Imagimg of reflectors under uncertainties in macromodel

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Introduction

A novel approach to imaging of layered media under uncertainties in a relevant macromodel has been developing. Method of Effective Specular Points (**MESP**) aims to get a clear depth image of complex interfaces from reflection data when transmission velocities are unknown. MESP exploits kinematical attributes of corporative effects of a pulse reflected by (oriented) elements of surface - rather than amplitudes, which are used to be interpreted as a result of superposition of waves "emitted" by (volumetric) diffractors. MESP assimilates unstacked reflection data from a rather sparse and irregular, in general, source-receiver net, say, $\{ x_s, x_r \}$, parametrized by lateral coordinates of source and receiver locations.
An illustration of problems which MESP is oriented to solve is given by Figure 4, `left`. A simple syncline results in caustics and related triplications (e.g., Figure 1), which is troublesomely even if transmission velocity is known: for traveltime tomography - because of multipathing of rays, and for migration/inversion - because of strong spatial variations of amplitudes. The latter is not a problem for the MESP as long as the method deals just with *kinematic* attributes of records, while the problem of multipathing is resolved *statistically:* we assume that first arrivals of pulses reflected by the reflector can be associated with their own elementary "mirrors", or *effective specular points* (**esp**'s). Some of **esp**'s can be in common use - similar to middle points in a vertically stratified medium, so the total number of **esp**'s is less or equal to that of source-receiver pairs. Thus the goal of MESP is following: **given** *estimates of multioffset traveltimes,* **find** *the most-focused depth image of a targeted reflector, while the focusing itself is governed by (uncertain) parameters of a macromodel of an overburden.* Mathematically MESP is posed as an optimization problem [3]: to find a velocity and such a reflector (a function of lateral coordinates), for which **esp's**, implied by estimates of traveltimes, have minimal variations with respect to the revealed reflector. In other words, we seek an optimal solution when every **esp** belongs to one of isochrones AND to the reflector. Note, that an **esp**, or *"mirror"* $\mathbf{m} = \mathbf{m}(\mathbf{x};\mathbf{n})$ has 3 and 5 parameters in 2D and 3D problems respectively: • $\mathbf{x}$ is radius-vector of *e.s.p.* location; • $\mathbf{n}$ is a unit normal indicating *e.s.p.* orientation. To "glue" all **esp** into one reflector we apply so called diffusion regularization, described by us earlier: [2]. This specific regularization allow us to get a differentiable surface of a virtual reflector, which makes necessary evaluations of gradients of relevant objective function to be fast and robust.
The approach is illustrated with processing synthetic (Figs. 1-4) and real data (Fig. 5).

Method and application

Let a set of observed traveltimes (see, e.g., Figures 1 - 3) constitute the input data set: $\mathbf{T}_{obs} = \{t_i,\ i = 1,\dots,I\}$, where index i corresponds to a source-receiver pair, and a symbol $\mathbf{M}$ stands for the entire set of **esp** locations/orientations: $\mathbf{M}^* = (\mathbf{X}^*,\ \mathbf{N}^*) = \{\, \mathbf{x}_i^*,\ \mathbf{n}_i^* \,|i = 1,\dots,I\, \}$ The problem is posed as following:

$$F(\,\varphi, \mathbf{M}; V\,) \to \min \tag{1}$$

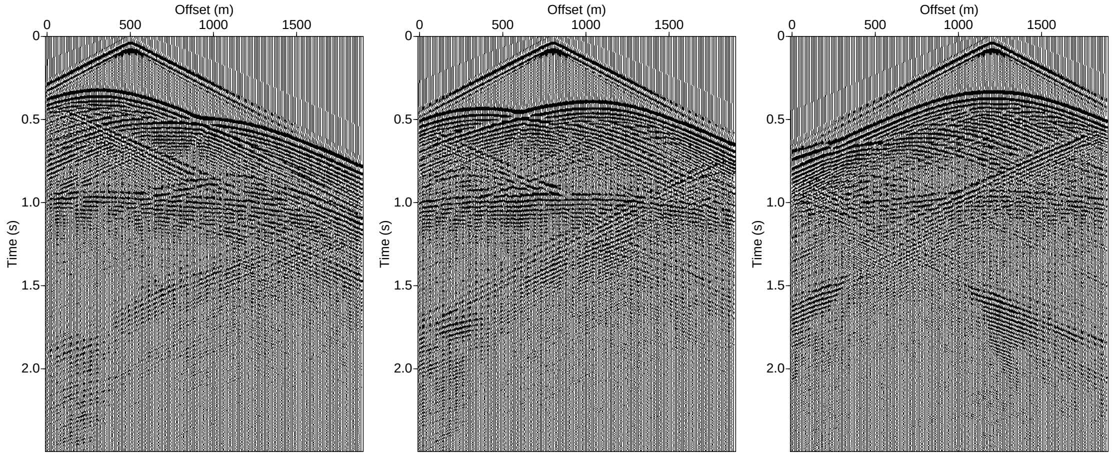

Figure 1: Three synthetic CSGs. Respective acquisition and reflector geometries are shown with Fig. 4 (`left`). Triplications, induced by the syncline, are well-seen.

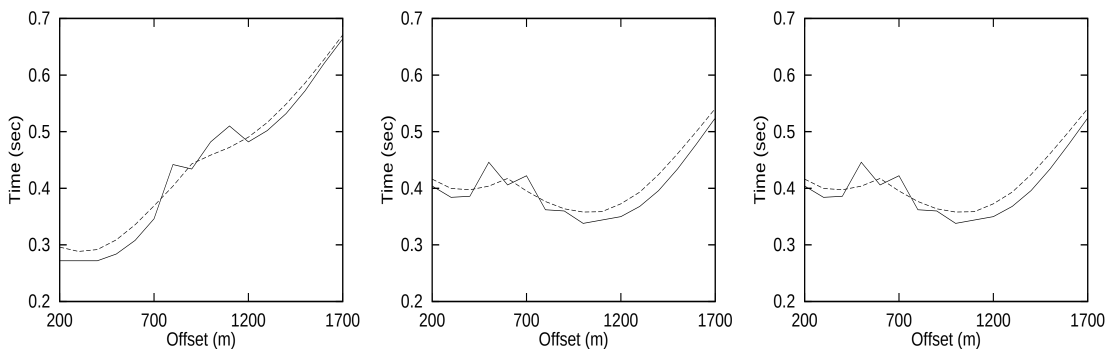

Figure 2: Related traveltime curves. Smooth ones represent times of traveling along Fermat's extremals, non-smooth ones are results of simulation of an automatic picking of first energy train arrivals. Note, triplications result in delayed arrivals.

where function $F = F_1(\mathbf{T}, \mathbf{X}, \varphi) + F_2(\mathbf{X}, \varphi) + F_3(\mathbf{N}, \varphi)$ (we omitted here the velocity-dependence of all components involved). The first term is a conventional for traveltime tomography weighted difference of observed $\mathbf{T}_{obs}$ and theoretical traveltimes $\mathbf{T}_{cal} = \{\tau_i(x_s^i; \mathbf{x}_i^*; x_r^i) = t_i + \varepsilon_i \mid i = 1, \dots, I\}$, where the latter is represented by a set of traveltimes along Fermat's extremals: $\{\, i - source \rightarrow \; i - esp \rightarrow i - receiver \mid i = 1, \dots, I \,\}$, while $\tau_i(x_s^i; \mathbf{x}; x_r^i) = t_i + \varepsilon_i$ as a function of free $\mathbf{x}$ gives a velocity-dependent family of isochrones: ε_i represents errors of estimated $\{t_i\}$. Thus the 1st term of the objective function (1) $F_1(\mathbf{T}, \mathbf{X}, \varphi) = \alpha_1 \sum_{i,j=1}^{I} C_{ij}^{-1}(t_i - \tau_i)(t_j - \tau_j)$ where C_{ij} is the covariance matrix of errors $\{\varepsilon_i\}$.
The 2nd positive function $F_2(\mathbf{X}, \varphi)$ yields a regularizing glue to gain all **esp**'s $\mathbf{M}$ located at $\mathbf{X}$ into one differentiable surface. $\| \varphi \|_3^2 = \alpha_2 \langle \varphi \mid e^{-\beta \nabla^2} \mid \varphi \rangle$, where positive parameter β controls a scale of reflector smoothness, which in its turn is induced by a size of the first Fresnel zone.
At last the 3rd term of the function (1) measures deviations of $\mathbf{N}$, "unit normals of **esp**", from normal vectors of current interface in the points with the same lateral coordinates ("Snell's law"). Introducing a set of related normals $\tilde{\mathbf{N}}$ we cam write the term in a form of a scalar product $F_3 = -\alpha_3(\mathbf{N}, \tilde{\mathbf{N}}) = -\alpha_3 \sum_i (\mathbf{n}_i, \tilde{\mathbf{n}}_i)$, where $\mathbf{n} = \nabla_{\mathbf{x}^*} \tau(\mathbf{x})$ and $\tilde{\mathbf{n}} = k\,(\,1, -\nabla_{x^*} \varphi(x))$ with $k = (1 + (\nabla\varphi\,, \nabla\varphi))^{1/2}$. . The function is minimal, when all pairs of normals coincide.

Method of optimization. The method of optimization of objective function is constructed as a sort of 'coordinate' descent: varying the weight factors α, one can get reduced optimization problems, that can be

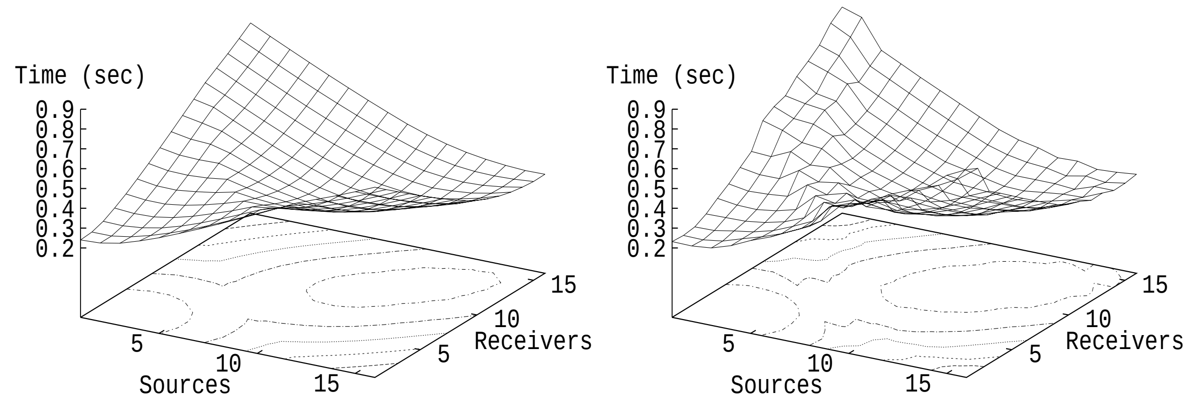

Figure 3: "First-arrival gathers" -traveltimes as a function of the source/receiver positions, 0-offset traveltimes are given with the values along diagonals. **Left:** theoretical travel times along Fermat's extremals; **right:** simulation of autopeaking. The problem is to map properly the surface of corrupted by autopeaking multi-valued traveltimes into the depth image of the reflector (Fig. 4), exploiting properly the extra dimension of data space.

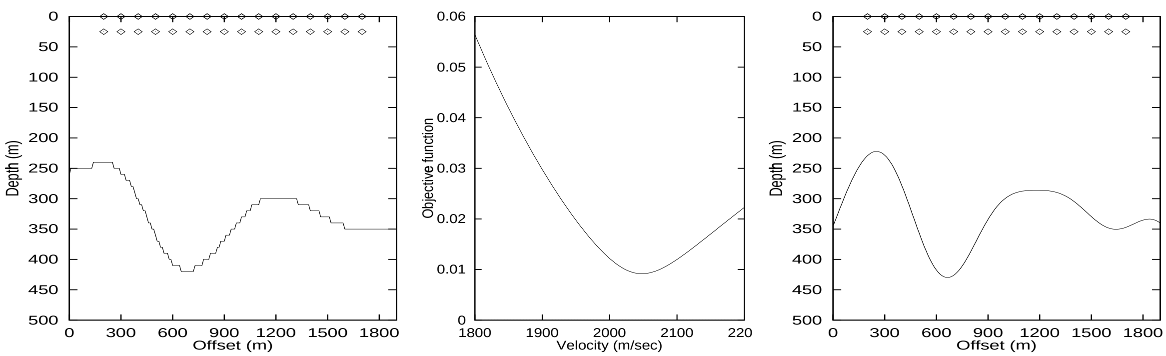

Figure 4: 2D acquisition and reflector geometry, objective function as a criterion of image focusing and the result of reconstruction. A 'true' location of the reflector (*left*) is given with a stepwise curve. Objective function on an interval velocity (a smooth convex curve *in the middle*) is a value of dispersion of e.s.p. depths with respect to an averaged interface (the 1st iteration). True velocity is 2000 m/s, estimated one is 2040 m/s. The resulting image after 3 iterations is displayed on the *right*.

solved much easier than the main problem (Eq.1). The first step consists of composing **esp**- set **M** with the deepest isochrone **esp**'s of fixed interval velocity (Snell's law is broken). This set gives the relevant regularized interface image, while the sum $F_1 + F_2$ as a function of velocity defines an *image focusing criterion.* Using "frozen" interface, gradients of $F_1 + F_3$ are evaluated, yielding the refining positions of specular points. Then iterations follow: the final images, reproduced with Fig. 4 for synthetic and Fig. 5 for real data, are the results of three iterative steps.

Real data. Here the approach is illustrated with simplified but well-verified experiment with real marine data: a sea-bottom is a target under artificial assumption that a speed of sound in the water is unknown. The problem is to find the speed and to get a depth image of a sea-bottom, using a sparse subset of seismic records: (21 locations of sources/receivers with $\Delta x = 150\ m$ on the segment of the line profile of 3000 meters (Fig.5). All other records have been used as a control subset: on the base of the reconstructed sea-bottom profile, the rms-misfit of traveltimes measured and those along Fermat's extremals/lines is evaluated - the decreasing of the misfit can testify for improving the imaging. The result is following:

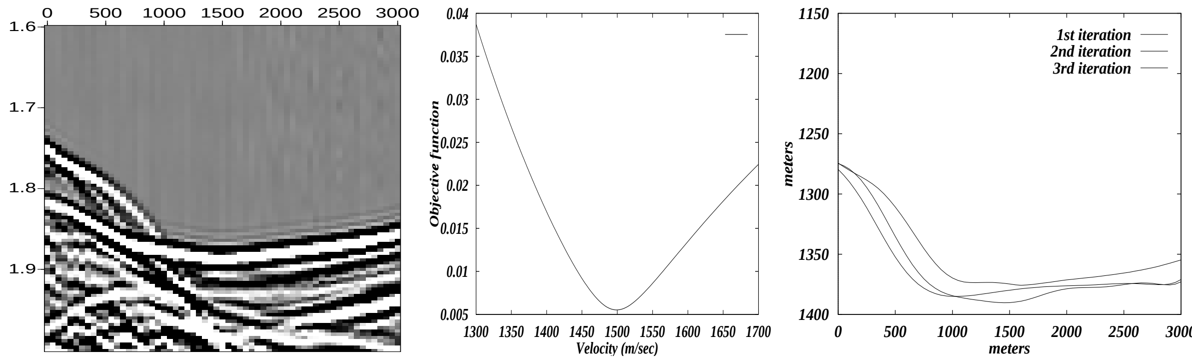

Figure 5: Segment of the "near 0-offset" gather (offset is equal to 101 m): *left*; objective function yields an estimate of the sound speed in water: 1499 m/s (in the *middle*), and 3 sequential **depth** images of the sea bottom are displayed on the *right*.

• the velocity estimated is 1499 m/s; • standard deviations of traveltimes are 0.032 sec for the 1-st image, 0.028 sec for the 2-nd and 0.025 sec for the final images respectively.
The simplified examples of processing synthetic/real data have been chosen here just to clarify the strategy of the method of Effective Specular Points: the imaging of a targeted reflector, when macromodel/reflectors of an overburden are given, needs in more sophisticated construction of isochrones.

Conclusions and discussion

The suggested MESP combines reflection traveltime tomography and depth migration/waveform inversion: instead of "superposition of waveformss" MESP exploits "superposition of isochrones" of a few-parameter family, while a single isochrone contributes in a reflector image just in a vicinity of its "own" effective specular point. Extra dimension of multioffset data (Fig. 3) results in revealing of vertical scale, providing us with a *depth* image. Statistical estimations of erronous prestack traveltimes need in a proper regularization and therefore result in a (slight) *bias* of the respective velocity estimate.
Kinematical attributes of reflected pulse are apparently dominant in imaging a reflector- different strategies give similar images provided they rely on the same kinematics. Besides, as long as a seismic reflection acquisition is of of a limited aperture and of an irregular, in general, location, a waveform inversion/migration leads to unavoidable artifacts - even in situation when caustics are negligible. Note here, that a strategy of a waveform inversion/migration, dependent on unknown many-parameter kinematics, was revealed by the authors of the abstract earlier [2], though a waveform inversion/migration is of much higher computational costs compared with the MESP. The latter can hopefully be very helpful at least for drastic reduction of the costs of iterative migration, while processing 3D-data sets [3] especially.

Acknowledgments

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