Eulerian-Eulerian simulation of sedimentation of uniformly-sized, non-Brownian spheres in viscous fluids

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Abstract. A Eulerian-Eulerian 3-dimensional CFD-model has been built in order to study the sedimentation process of uniformly-sized, non-Brownian spheres in a viscous fluid. It is shown that proper agreement with experimental data for high particle loadings strongly depends on using a correct drag force relation. Convection currents in the carrier phase were observed during simulation. They were found to present a hydraulic resistance to the solids settling. The particle interactions were implemented through the solid stress tensor dependent on the viscosity of the carrier phase and the solids local volume fraction. The completed model was validated by experimental data with discrepancies of 5-10%.

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INTRODUCTION

Solids sedimentation finds many applications in industry as well as in science. Even though this phenomenon has been well studied, the full set of underlying mechanisms is not completely understood [1]. One of the key parameters which characterize settling is the particle terminal velocity. It is achieved when the solid acceleration becomes zero and the particles move uniformly downwards. For a single non-Brownian particle the terminal velocity is the Stokes velocity and it depends on the fluid parameters (density, viscosity) as well as on the particle density and diameter [2]. When a cloud of particles settles in fluid the terminal velocity becomes lower than the Stokes one. It has been empirically found that the reduction in terminal velocity depends on solids concentration in the system. One of the commonly used corrections for this parameter can be found in Richardson and Zaki [3]. Reviewing empirical observations of settling particle clouds 4 different mechanisms for slowing the process down can be mentioned:

1. According to Crowe et al. [4] the drag force is modified by particle concentration and becomes stronger than for a single particle. This increases the fluid resistance to particle movement.
2. During settling, particles collide and induce collisional stress inside the solid phase. Moreover, due to collisions the particle velocity vectors will be dissimilarly orientated, which may cause velocity fluctuations in vertical and horizontal directions [2] and, as a result, the settling velocity becomes lower.
3. When a significant amount of solids sinks to the bottom of the fluid it pushes some liquid up due to incompressibility of both phases.
4. In cases of cross-sectional non-uniformity regions of faster and slower upward fluid flow will exist,with attendant non-uniformities in the particle flow. Mechanisms (3) and (4) create gulf-stream patterns of fluid [5].

The present work simulates the process of solids settling for a dense suspension of nearly uniformly-sized particles at low Reynolds number. The drag force for high particles loading was evaluated and tested. The solids stress was incorporated in the model, and the phenomenon of gulf streaming was modeled and investigated.

MODEL

The simulated process repeats the experiment of Nicolai et al. [2]: 800 μm glass beads settled under gravity in 500×100×400 mm vessel filled with viscous fluid. The solids volume concentration was varied up to 40% and the mean settling velocity was found as a function of the solids volume fraction. A numerical model was built in the commercial package STAR-CD. The governing equations for the 2-phase model performed in 3 dimensions are given below. The phases are treated using Eulerian-Eulerian technique which assumes the solids behave liquid-like at high concentrations:

$$\frac{\partial(\alpha_m\rho_m)}{\partial t}+\nabla(\alpha_m\rho_m\vec{u_m})=0 \tag{1}$$

In Equation (1), index m assigns the phase (liquid or solid), α is the volume fraction, ρ is the density and $\vec{u}$ is the mean phase velocity. The momentum equation is given by:

$$\frac{\partial(\alpha_m\rho_m\vec{u_m})}{\partial t}+\nabla(\alpha_m\rho_m\vec{u_m}\vec{u_m})=-\alpha_m\nabla p+\alpha_m\rho_m\vec{g}+\nabla(\alpha_m\tau_m)+\vec{M_m}+\vec{F_A} \tag{2}$$

where p is the pressure, $\vec{M}$ is the inter-phase momentum transfer term, $\vec{F_A}$ is the buoyant force applied only to the solid phase, τ is the molecular stress, $\vec{g}$ is the acceleration due to gravity. The inter-phase momentum transfer is coupled $M_l=-M_s$ where l and s are for liquid and solid phase respectively. The molecular stress for both phases is presented in Cartesian coordinates as follows:

$$\tau_m^{ij}=2\mu_m D^{ij}-\frac{2}{3}\mu_m\delta^{ij}\nabla\vec{u_m} \tag{3}$$

where i,j designate the coordinates, μ is the dynamic viscosity kept constant for the liquid phase, δ^{ij} is the Kronecker delta, $D^{ij}=\frac{1}{2}(\frac{\partial U^i}{\partial x^j}+\frac{\partial U^j}{\partial x^i})$. The energy equation for both phases was not considered due to low energy change in the system. Since implementation of granular stress [6] was accompanied with high computational cost, the term τ for the solid phase was modeled through a user subroutine for an apparent suspension viscosity $\mu_s=\mu_l(1-\varphi)^{-2.5}$ according to Roscoe-Brinkman [10], where φ is the local volume fraction of solid phase per finite volume.

The inter-phase momentum transfer term $\vec{M}$ was presented by the drag force $\vec{F_D}$. Although a lot of formulations have been proposed for the solid-fluid drag mechanism, a universal one, which would model the drag interaction for a wide range of solids concentration, does not exist. In the present work the drag force expression by Ergun [7]:

$$F_D=\frac{1.75\alpha_s\rho_l Vr^2}{d}+\frac{150\cdot\alpha_s\cdot\mu_l Vr}{\alpha_l\cdot d^2} \tag{4}$$

and Gosman et al. [8]:

$$F_D=\frac{3}{4}\frac{\alpha_s\rho_l C_D}{d}Vr^2 \tag{5}$$

were compared analytically to experimental data in order to select the most suitable. In Equations (4) and (5) d is the particle diameter, Vr is the relative velocity (equal to solids settling velocity in static fluid) and C_D is the particle drag coefficient calculated according to Shiller-Naumann [8].

Spatial discretization of governing equations was done by the finite-volume technique with 31250 polyhedral control volumes, the time advancement was perfomed implicitly. Pressure-implicit splitting of operator algorithms (PISO) [9] was used to solve the equations numerically. Initial condition was zero velocity field and atmospheric pressure all over the computational domain. The boundary condition used was no-slip wall for both phases, and the flow was assumed to be laminar.

RESULTS AND DISCUSSIONS

The equation $F_D=(\rho_s-\rho_l)g$ was solved theoretically for the terminal velocity in order to compare the drag force expressions by Ergun (Equation (4)) and Gosman (Equation (5)) to experimental data. The results are presented in Figure 1A. It was concluded that the terminal velocities calculated using the Gosman expression were higher than they should be in reality. Thus the drag force was too weak to slow the settling properly. Ergun derivation agreed quite well with the experimental data, hence it was used for further model development.

The first simulations were performed with inviscid solid phase. This way the shear stress (Equation (3)) in solids was equal to zero and presents the case where inter-particle interactions are neglected. The simulation results are shown in Figure 1B and it can be seen that they are close to the experimental curve. The discrepancies to experimental data were in the interval 8-41%.

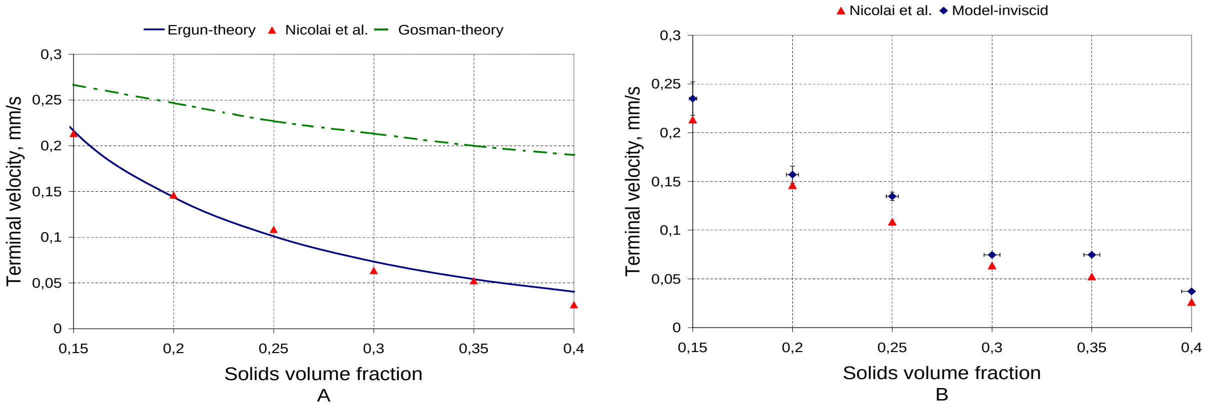


FIGURE 1. Drag force and solids stress evaluation. A - terminal velocity against solids volume fraction for theoretical calculations by Gosman and Ergun compared to experimental data. B-terminal velocity against solids volume fraction for CFD-model (inviscid solid phase) compared to experimental data

Physically inter-particle interactions existed in the solid phase, which influensed the process. During transient analysis it was found that convection patterns were formed in the liquid phase. In Figure 2 it is shown that the solids volume fraction profiles are semi-centric to the patterns, and that the mean settling velocity V is dependent on the size of the pattern. This phenomenon was quantified in terms of the hindered settling function $\frac{V}{V_{Stokes}}$ and the pattern geometrical parameter D/L. As it is shown in Figure 2, D is the recirculation pattern width (mean in case of several patterns) and L is the distance between the sedimentation front and the pattern (mean for several). The dependence (Figure 3A) between the two described parameters is close to linear. The settling velocity decreases as the gulf-stream pattern expands.

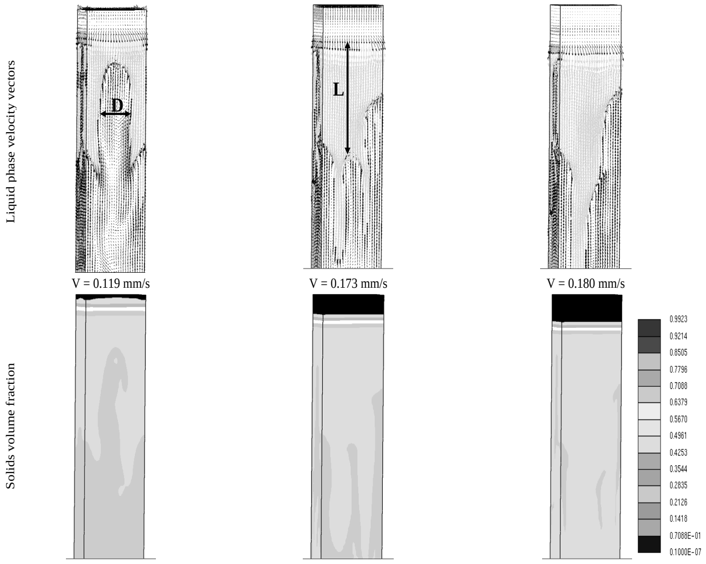
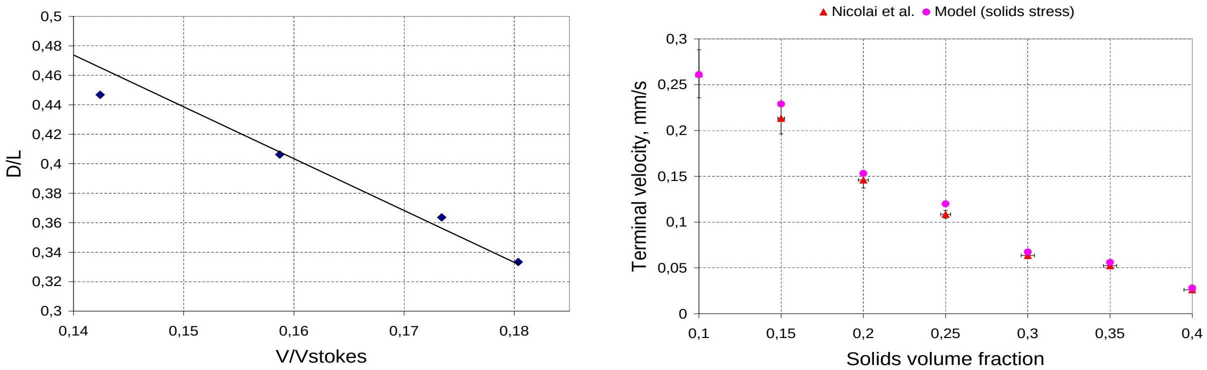


FIGURE 2. Liquid phase velocity vectors, the contours of solids volume fraction for different mean sedimentation velocities (initial solids volume fraction 35%)

The solids stress (Equation (3)) was modeled by implementing a user subroutine for the solid phase according to the Roscoe-Brinkman relation [10]. The simulation results, shown on Figure 3B, are within the experimental error bars. The discrepancies to experimental data were in the interval 5-10%.



A

B

FIGURE 3. Convection pattern characteristics and completed model results. A - convection current geometrical parameter (dimensionless) against hindered settling function. B - terminal velocity against solids volume fraction for CFD-model (solids stress) compared to experimental data

CONCLUSIONS

Eulerian-Eulerian simulations of settling particles have been performed. It is shown that the drag force expression by Ergun (Equation (4)) correlates properly with the hindering effect. Convective currents in the suspension were observed and quantified. They were found to constitute a hydraulic resistance to the solids settling. The mean sedimentation velocity depends on the geometrical parameter of the gulf-stream pattern. The solids stress was implemented through an expression for apparent viscosity of a suspension. The completed model results were in good agreement with experimental data.

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