Substorm and magnetosphere characteristic scales inferred from the SuperMAG auroral electrojet indices

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[1] A generalization of the traditional 12-station auroral electrojet (AE) index to include more than 100 magnetometer stations, SME, is an excellent predictor of global auroral power (AP), even at high cadence (1 min). We use this index, and a database of more than 53,000 substorms derived from it, covering 1980–2009, to investigate time and energy scales in the magnetosphere, during substorms and otherwise. We find, contrary to common opinion, that substorms do not have a preferred recurrence rate but instead have two distinct dynamic regimes, each following a power law. The number of substorms recurring after a time \( \Delta t \), \( N(\Delta t) \), varies as \( \Delta t^{-1.19} \) for short times (<80 min) and as \( \Delta t^{-1.76} \) for longer times (>3 hours). Other evidence also shows these distinct regimes for the magnetosphere, including a break in the power law spectra for SME at about 3 hours. The time between two consecutive substorms is only weakly correlated (\( r = 0.18 \) for isolated and \( r = 0.06 \) for recurrent) with the time until the next, suggesting quasiperiodicity is not common. However, substorms do have a preferred size, with the typical peak SME magnitude reaching 400–600 nT, but with a mean of 656 nT, corresponding to a bit less than 40 GW AP. More surprisingly, another characteristic scale exists in the magnetosphere, namely, a peak in the SME distribution around 61 nT, corresponding to about 5 GW precipitating AP. The dominant form of auroral precipitation is diffuse aurora; thus, these values are properties of the magnetotail thermal electron distribution. The characteristic 5 GW value specifically represents a preferred minimum below which the magnetotail rarely drops. The magnetotail experiences continuous loss by precipitation, so the existence of a preferred minimum implies driving that rarely disappears altogether. Finally, the distribution of SME values across all times, in accordance with earlier work on AE, is best fit by the sum of two distributions, each normal in \( \log(SME) \). The lower distribution (with a 40% weighting) corresponds to the characteristic quiet peak, while the higher value distribution (60% weighting) is an average over the characteristic substorm peak and the subsequent prolonged recovery.


1. Introduction

[2] Anecdotally substorms recur every few hours, releasing a few hundred megajoules and creating depressions of a few hundred nanotesla in the auroral electrojet (AE) AL index. A finite number of observations, each of a finite value, will always yield a “mean” time between substorms and a mean magnitude. It is not necessary that these values be preferred (or, as a matter of fact, necessarily even physically meaningful). An exponential distribution has a well-defined characteristic scale (the e-folding magnitude) but no preferred value.

There is some evidence both for and against preferred scales in the magnetosphere. For example, Borovsky et al. [1993] used the occurrence rate of dispersionless energetic particle injections as observed by geosynchronous satellites to infer a preferred time between substorms of about 2.75 hours, with an additional exponential distribution with a characteristic 5 hour e-folding scale at longer intervals. Likewise, Lui et al. [2000] examined the size and power associated with auroral blobs, to find that during times including substorms a preferred size for auroral blobs exists. Vassiliadis et al. [1996] and Consolini and De Michelli [1998] have found that the distribution of AE and AL values is well fit by the sum of two distributions, each normal in \( \log(AE) \) or \( \log(AL) \). The latter results imply the existence of both a quiet time and an active time characteristic values for these indices.

[3] Most examinations of global geomagnetic indices have not shown preferred time scales. Although some work on very small sets of data showed a possible 3 hour periodicity in the AE index [Shan et al., 1991], more comprehensive
work using a year or more has not evidenced any particular spectral power at 3 hours or in fact any time shorter than 24 hours [Tsurutani et al., 1990]. In fact, regarded as a time series, \( AE \) can be characterized as colored noise (\( f^{-2} \)) with two regimes of distinct exponents [Takalo et al., 1994]. Similarly, Tsurutani et al. [1990] found the spectral break in the \( AE \) power to occur around 3 hours. This spectral break has reasonably been associated with the substorm cycle [Rypdal and Rypdal, 2010], as that is about the time involved in a growth phase, expansion, and subsequent recovery. At times longer, the \( AE \) distribution approaches stationarity [Rypdal and Rypdal, 2010].

[4] With magnetic indices, the question of physical meaning often arises. The interpretation of \( AE \) itself has often been regarded as particularly problematic. Thus Kamide and Rostoker [2004] stated “We would like to propose ceasing the derivation and distribution of \( AE \) index because it has no physically interpretable meaning.” Recently, we have demonstrated that the \( SME \) index, which is the SuperMAG generalization of the traditional \( AE \) index to include more than 100 magnetometer stations instead of the usual 12, is a remarkably good characterization of the global auroral power (\( AP \)), even when examined at 1 min cadence. Thus, the nightside \( AP \) associated with a Polar UVI image can be predicted with a correlation coefficient of \( r = 0.86 \) using the 1 min \( SME \) [Newell and Gjerloev, 2011]. The hourly correlation rises to \( r = 0.88 \), and this is done without even accounting for seasonal effects. Even \( AE \) correlates at the 0.81 level (1 min cadence). It is worth noting that \( SME \) correlates better with both premidnight and postmidnight power than does either \( SML \) (putatively the westward \( AE \), typically postmidnight) or \( SMU \) (putatively the eastward \( AE \), typically premidnight). Thus, \( SME \) (and, to a lesser extent, \( AE \)) has, in fact, a particularly clear geophysical meaning. As most of the \( AP \) is from diffuse aurora, specifically electron precipitation [Newell et al., 2009], \( SME \) is a sampling of the thermal component of the magnetotail electron population.

[5] The more global coverage of \( SME \) makes it possible to define substorm onsets using \( SML \), the 100+ magnetometer station generalization of \( AL \), that agree rather well with global imaging [Newell and Gjerloev, 2011]. The availability of many years of this more geophysically meaningful index at 1 min cadence leads us to reconsider the question of characteristic scales in substorms. We find that, indeed, substorms do have a preferred magnitude, namely, an \( SME \) mean of about 656 nT, with a modal peak in the 400–600 nT range. The closely correlated implied \( AP \) is 38 GW for the mean substorm peak \( AP \) (based on Defense Meteorological Satellites Program (DMSP) calibration, which is slightly lower than for the Polar UVI calibration). However, we do not find evidence for any preferred timescale in the substorm recurrence, finding instead a power law (scale-free) distribution with two dynamical regimes, one for recurrent substorms (within 80–90 min or less of a previous substorm) and one for isolated substorms (>3 hours). We argue that this result is more satisfactory than the current standard understanding. If substorms had both a preferred size and a preferred recurrence rate, they would produce a more or less steady output energy. This is inconsistent with the irregular and largely scaleless nature of solar wind driving. It appears instead that substorms of a roughly standard size recur at whatever irregular interval is necessary to retain balance with irregular solar wind driving.

2. Data and Methods

2.1. The SuperMAG \( SME \) (\( SMU \), \( SML \)) Indices

[6] The \( AE \) indices were introduced using five magnetometer stations by Davis and Sugiura [1966], based on the \( H \)-component (aligned toward local geomagnetic north for each station) and include an upper envelope (\( AU \)), a lower envelope (\( AL \)), and the difference \( AE = AU – AL \). It is thought that \( AU \) represents the strength of the eastward \( AE \), primarily in the dusk cell. \( AL \) represents the westward \( AE \), primarily in the morning. During substorm onset, however, the station contributing to the lower envelope is usually in the dusk sector beneath the auroral expansion [e.g., Gjerloev et al., 2004]. The original 5-station \( AE \) index was soon extended to 12 stations, and the latter value has been the standard for the last three decades. The limitations of using so few stations for such a global index are well known [e.g., Rostoker, 1972]. However, Kamide et al. [1982] used about 70 magnetometer stations to derive \( AE(70) \) at a 5 min cadence for a limited time (selected days). It seems quite reasonable to assume that \( AE(70) \) is better than \( AE(12) \), but Kamide et al. [1982] did not investigate the issue quantitatively. Newell and Gjerloev [2011] used SuperMAG to introduce \( AE \) indices at a 1 min cadence for multiple decades using up to 130 stations. Because the 12-station \( AE \) index is an official International Association of Geomagnetism and Aeronomy (IAGA) product, an alternate name, \( SME = SMU – SML \) is necessary for the SuperMAG version. Nonetheless, conceptually \( SME \) may be regarded as \( AE(100) \).

[7] The widespread usage of \( AE \) has also been hampered by the difficulty in obtaining it at high cadence (say 1 min) for most years. Both issues are now satisfactorily addressed. Newell and Gjerloev [2011], using global images from Polar UVI, showed that \( AE \) is highly correlated with \( AP \) and, in fact, predicts that power much better than does \( AU \), \( AL \), \( Kp \), or any known solar wind driving function. It turns out that a substantial further improvement in predictive power occurs when \( SME \) replaces \( AE \). Thus, for example, the \( AP \) from Polar UVI “instantaneous” images (typically 1 min) on the nightside correlate with \( Kp \) at \( r = 0.71 \) level, correlate with traditional \( AE \) at \( r = 0.81 \) level, and correlate with \( SME \) at \( r = 0.86 \) level (thus representing a 9% further improvement in variance accounted for). \( SME \) probably does not predict dayside \( AP \) as well, but neither does dayside power significantly participate in substorms [Newell et al., 2001]. Thus, \( SME \) (and indeed, even traditional \( AE \)) are much more physically meaningful than previously supposed. About three decades worth of \( SME \) at a 1 min cadence are available from the SuperMAG websites.

2.2. Substorm Database

[8] Our database consists of somewhat more than 53,000 substorms from 1 January 1980 to 31 December 2009, identified from the \( SML \) index. The details about identifying substorms from \( SML \) are given in the study by Newell and Gjerloev [2011], along with considerable verification versus satellite data. Therefore, we give here only a brief summary. The substorms were defined by a drop in \( SML \) that was sharp (45 nT in 3 min) and that was sustained (−100 nT
average for 25 min starting 5 min after onset). The timing of the SML-identified substorms agrees reasonably well with Polar UVI (although with a median lag of about 4 min). The SML-identified substorms have the same characteristic and sustained increase in global AP as to Polar UVI-identified substorms [Newell and Gjerloev, 2011]. This represents a considerable improvement on using the traditional AL(12) to identify the substorms, as was also shown in that article.

[9] We also used DMSP data in a superposed epoch analysis study to investigate specifically isolated versus recurrent substorms, the latter identified as those within 2 hours of the previous onset. Recurrent substorms prove to be recurrent substorms, the latter identified as those within 2 hours. In fact, the increases in AP for each type of aurora were the same (e.g., about 8 GW for diffuse aurora and 1.5 GW for broadband aurora) for isolated and recurrent substorms. Thus, a substorm identified by SML represents an abrupt and sustained increase in global auroral precipitating energy flux. This corresponds well with the original definition of an auroral substorm.

2.3. Relation Between SME and AP

[10] We have calibrated the SME with both Polar UVI instantaneous images (actually typically 1 min) and with DMSP precipitation maps (essentially one precipitation map is constructed for each 5 nT step in SME, with separate maps for each type of aurora). Both sets of auroral data imply SME is approximately linear in total AP. The correlations between SME and both DMSP and Polar UVI are quite high. However, only Polar UVI measures AP on an instantaneous basis and thus provides the most meaningful correlation coefficients. Still, while the Polar imagee combines all types of aurora, with only the total power available, the DMSP calibrations allow us to see how each type of aurora separately varies. The result of interest here is that AP is given in gigawatts by

\[
AP = 0.037 \times SME\text{(diffuse)} + 0.005 \times SME\text{(broadband)} + 0.006 \times SME\text{(ions)} + 0.241 \times SME^{1/2}\text{(monoenergetic)},
\]

or, simplifying,

\[
AP = 0.048 \times SME + 0.241(SME)^{1/2}.
\]

It is the monoenergetic aurora that contributes the square root dependency. If the calibration with Polar UVI were used, higher power would be predicted for moderate and higher SME, mostly because of a calibration difference between Polar and DMSP (we do not know which is more accurate, using NOAA satellites would give still a third result). Total power is quite linear with SME in Polar observations. Polar UVI does not observe low levels of diffuse aurora very well, if at all, and thus has an offset, with the image power going to zero around SME = 50 nT. However, the more sensitive DMSP observations do not show the global AP dropping to zero for finite SME. Instead, power declines smoothly to slightly above zero at SME = 0 nT. Because of its longevity, superior sensitivity, and detailed spectral measurements, we choose here to use the DMSP calibrations to convert SME to AP. However, using the two-term formula above does not improve predictive correlation with Polar UVI (total AP) over using a single-term linear in SME.

3. Characteristic Scales for Substorms

3.1. Characteristic Recurrence Rate

[11] The time between consecutive substorms was calculated over the entire database of more than 53,000 substorms. Figure 1a shows the distribution of the time interval between subsequent substorms sorted into 10 min bin steps. As there are no gaps in SME data in any year used, there likewise are no gaps in the record of onsets. The mean time between substorms in our database is 265 min (about 4.4 hours), which is moderately smaller than the 5.75 hour mean reported by Borovsky et al. [1993]. The primary difference with the work based on geosynchronous particle injections is that the latter reported virtually no substorms recurring on times shorter than about 2.75 hours. In fact, it is difficult to identify a second dispersionless injection of energetic particles shortly after the first. In the present set of observations, the recurrence rate rises to the lowest time bin available (30 min). We do not go below this because our selection time requires both a sharp drop in SML, and that the drop is sustained for 25 min (starting 5 min after the drop). This selection requirement on a sustained drop in SML would not allow a meaningful identification of substorms or a shorter timescale.

[12] Figure 1b shows the log of the recurrence rate. A least squares fit is shown. The best fit implies that the number of substorms recurring at time t varies as \(N(t) \sim e^{-t/4.8}\). This is close to the value reported by Borovsky et al. [1993] for an exponential fit, namely, 5 hours. As previously mentioned, however, we find no dropoff of substorms below 3 hours. Moreover, the fit to an exponential is not particularly convincing.

[13] On the basis of a suggestion by K. Rypdal (personal communication, 2011), we tried a log-log plot (using the same 10 min bin size) to investigate a possible power law dependence. Figure 2 shows that the suggestion was insightful. In fact, there are two clearly defined power law regimes. At shorter times (up to the bin centered at 82 min), we find \(N(t) \sim t^{-1.19}\). At longer times (3 hours and beyond), \(N(t) \sim t^{-1.76}\). There should be little doubt that the power law fits shown in Figure 2 are far more convincing than is the exponential fit of Figure 1. Between about 1.5 and 2.5 hours, the curves smoothly transition between the shorter time and longer time behaviors. This suggests the existence of two distinct dynamical regimes, one for recurrent substorms and one for isolated substorms.

[14] To test robustness, we rebinned the data to a 20 min size. The same two dynamical regimes appear but with exponents of \(-0.95\) and \(-2.0\) (based partly on manually changing the breakpoint used for making the fits to 2.5 hours). This gives some indication of the uncertainties in the slope fits. However the plot shown in Figure 2 uses the breakpoint (82 min), which has the tightest fit to the data.

[15] The power law form for substorm recurrence does raise questions about the physical significance of the observed mean time between substorms, as was pointed out to the authors by K. Rypdal (personal communication, 2011). We will defer discussion of that and related issues for future work, except to note that, of course, the magnetosphere must...
“know” how long it has been since the last onset, else a Poisson process and an exponential form would result.

[16] Because the previously reported peak at about 3 hours is widely interpreted as a periodic component to substorm recurrence, we examine the correlation between one intersubstorm time interval and the next. That is, if three consecutive substorms occur at epochs $t_1$, $t_2$, and $t_3$, we consider $t_3 - t_2$ as a function of $t_2 - t_1$ (the latter being treated as the independent variable and plotted on the $x$ axis). Given that we have 53,000 plus substorms, and the correlations are weak, a scatterplot is not helpful (except possibly to graphically demonstrate the weakness of the correlation). Figure 3 plots the relationship

**Figure 1.** (a) The distribution of intervals between consecutive substorms. (b) The natural log of the number of substorms versus interval between substorms. The distribution is not well fit by an exponential, even neglecting shorter timescales.
**Figure 2.** The number of substorms recurring after a time $\Delta t$, on a log-log scale. The result is two power law regimes, one for time $\leq 82$ min, and one for time $>3$ hours ($5.2$ on this scale). The fit is markedly better than was an exponential (Figure 1).

**Figure 3.** Testing whether recurrence times between adjacent substorms are correlated. Thus, if three substorms occur at $t_1$, $t_2$, and $t_3$, the $x$ axis is $t_2 - t_1$, while the $y$ axis is $t_3 - t_2$ (although the data are binned). The correlations are weak, albeit statistically significant.
between one intersubstorm interval and the next, using 10 min bins for the $x$ axis. Because of the large number of points in each bin, information about the mean is improved (the standard deviation of the mean is reduced by the square root of the number of points), despite the large scatter in the sample. In fact, there is a slight tendency for the time between two substorms to predict the time interval until a third. For shorter times, up to about 1.5 hours, the correlation is extremely weak, only $r = 0.06$, thus explaining only $r^2 = 0.36\%$ of the variance. At longer times, however, the correlation improves to a still modest $r = 0.18$ (explaining 3.2% of the variance). While not denying that the correlation is high enough to be physically meaningful, our interpretation is that quasiperiodic sequences of substorms (such as the so-called sawtooth events) must be comparatively rare. In any case, Figure 6 does provide further evidence of two distinct dynamical regimes, with recurrent substorms.

Figure 4. The distribution of the 15 min average magnitude of (top) SML and (bottom) SME (starting 10 min after onset). Substorms have a well-pronounced characteristic size, just below 40 GW, based on DMSP cross calibration.
(<1.5 hours or so) behaving differently than isolated substorms (>3 hours).

[17] In principle, Figure 3 conveys information about the extent to which information about the state of the magnetosphere survives onset. Trying to fully exploit this information is outside the scope of this study. The break around 90 min again implies the existence of distinct dynamical behaviors for the magnetosphere shortly after a substorm and for isolated substorms.

3.2. Characteristic Size

[18] As hemispheric $AP$ is highly correlated ($r = 0.86$) with the 1 min SME averages, the magnitude of the latter provides one good measure, perhaps the most logical, of substorm magnitude. However, substorms are more commonly characterized by the magnitude of $AL$. Thus, Figure 4 shows the distributions of both the magnitude of $SML$ and SME (the bin width is 20 nT for the former and is 30 nT for the latter). These values are calculated as a 15 min mean, commencing 10 min after onset is identified. That time interval, from 10 to 25 min after onset, is about the time it takes to reach the $AP$ peak in global imaging superposed epoch analysis plots [Newell et al., 2001], but a little less than the time needed from particle superposed analysis [Newell et al., 2010], which indicates that the diffuse aurora takes 30–35 min to peak. Nonetheless, it is a reasonable standard range near the peak of precipitating power in a typical substorm.

[19] Figure 4 clearly shows that substorms do have a characteristic magnitude scale. In an analysis of auroral “blobs” around storm onset time, Lui et al. [2000] previously found evidence for a preferred size and power associated with substorms. Analyzed as in Figure 4, the evidence is striking. Despite the presence of a high magnitude tail, there is a very well-defined peak at about 375 nT for $SML$. Similarly, there is a clear modal peak between 400 and 600 nT for SME, with a mean of 656 nT. Using the previously determined correlation between SME and hemispheric power, the mean corresponds to just less than 40 GW peak power expenditure. (Again, we used calibrations with DMSP here to convert SME to $AP$, and slightly different, typically higher, number would result from using a Polar UVI-based calibration.)

[20] It may be questioned whether the minimum SME is inadvertently imposed by the selection criteria on $SML$ for identifying substorms. Our basic criteria are a sharp drop (45 nT in 3 min) in $SML$, which is sustained (100 nT average drop from the initial value over 25 min). Therefore, drops less than 100 nT are largely excluded. To test the sensitivity of the criteria, we reran the substorm identification algorithm using a requirement of just 50 nT drop, and a third time requiring a steeper 200 nT drop. Although the imposed 100 nT threshold does seem to somewhat truncate the curve (compared to the alternate 50 nT requirement), and the mean shifts slightly higher for higher minimum thresholds, the essential fact remains that a peak exists at about the same value (or range of values, as the peak is broad), with the same dropoff above 500 nT. Figure 5 shows the distribution of substorm sizes under the most minimal requirement (50 nT sustained drop) reasonably consistent with the condition of having a drop in $SML$ at all. A characteristic peak for SME is thus not imposed by selection, although the computed mean value is affected if smaller events are

![Distribution of Substorm SML Magnitude](image)

**Figure 5.** The database was reexamined for substorm onsets using an alternate lower threshold for $SML$ magnitude (50 nT). Even under this probably too low value, substorms still have a characteristic magnitude extending to about 500 nT, as is true with lower and higher thresholds.
4. SME Spectral Power and Distribution

In section 3, we considered scales characteristic of substorms. It is quite useful to consider all SME data in ensemble and look for scales that apply across the entire dynamic range of the magnetosphere, irrespective of the substorm phase. It turns out that the results are quite similar to the considerable previous work done on AE spectral power and scaling [Tsurutani et al., 1990; Takalo et al., 1994; Vassiliadis et al., 1996; Consolini and De Michelis, 1998; Rypdal and Rypdal, 2010] as discussed in the following. See also reviews by Klimas et al. [1996] and Freeman and Watkins [2002].

4.1. Spectral Power

We calculated the spectral power in each of SME indices for each of 10 years from 1997 to 2006. As the results were broadly similar from year to year, we averaged together the spectral power over these years to produce Figure 6, the distribution in relative power from 6 months to a few minutes. The “y” axis is the absolute magnitude squared of the transform coefficients at each frequency. The SME data sampling period ($T = 1$ min) was preserved in performing the Fourier transform calculation. Figure 6 shows a pronounced peak at 24 hours, a small peak at 9 days, and a spectral break in the slope around $10^{-4}$ Hz (corresponding to a period of about 3 hours). At frequencies above that peak, the fall off is essentially featureless, or so-called colored noise, with a $f^{-1.6}$ spectral behavior. There is no peak at 3 hours in the spectral distribution. The spectral behavior of SML and SMU is similar to SME, although the slopes differ slightly. SMU shows a peak at 12 and 24 hours. These results all agree generally with a previous work on the spectra of AE. For example, both
Tsurutani et al. [1990] and Takalo et al. [1994] reported similar results. There are a few differences. Both sets of earlier authors reported the spectral break in AE at about 5 hours, while SME exhibits a break at about 3 hours, which indeed is the approximate time the behavior of substorms changes (Figures 1–3). Indeed, Rypdal and Rypdal [2010] previously argued that the spectral break should correspond to a substorm behavior, whose suggestion certainly strikes us as logical, and therefore, the 3 hour spectral breakpoint may be more consistent. As the last named authors also pointed out, the behavior of SME tends toward stationarity at longer time intervals. However, that approach is gradual, and there is first an intermediate regime corresponding to the isolated substorm regime in Figures 2 and 3.

In summary, the fundamental description of AE as "bicolored noise" given by Takalo et al. [1994] does not seem to change with the introduction of the SME index. We will therefore not explore the issue further here.

4.2. The Distribution of SME Values

As SME is intimately connected with hemispheric AP, the distribution of SME across all conditions is a rather good way to examine global precipitating power over a variety of timescales over many years. It might be the only way currently possible. Aside from the fact that global imagers typically view only a portion of the auroral oval for a part of a day, they are also not sensitive to modest energy fluxes below about 0.25 ergs cm$^{-2}$ s, which can still amount to several gigawatts of power integrated over the globe. As the diffuse aura is the dominant source of AP [Newell et al., 2010], the distribution should largely reflect the energy density in the magnetotail as a whole, or, more properly, the thermal electron population, which has a typical temperature of about 1 keV. Figure 7 shows the SME distribution in 1 nT bins, from SME = 0 to 500 nT. Although substorms can easily exceed this, we verified that nothing interesting happens in the distribution as it is extended to 1500 nT. Therefore, we prefer to plot the data on a scale that shows the most interesting feature, namely, the clear peak at a modal value of SME = 61 nT. This is about 10% of the substorm peak and thus represents a separate quiet time peak. The peak corresponds to approximately 5 GW of power (from equation (2)). On the basis of the formulas given in equation (1), as well as in the previous work [Newell et al., 2009; Thorne et al. 2010], that power will be about 80% diffuse auroral precipitation, with monoenergetic aura a distant second (followed by roughly equal broadband or wave aura and the ion aura). Under the assumption of strong pitch angle scattering maintaining the plasma sheet loss cone, this characteristic peak in the distribution represents a preferred quiet time electron energy density in the magnetotail. Most of the energy density in the magnetotail is actually stored in ions, not electrons. However, typically, the electrons have the same density and about 1/5 the temperature as the ions. Given that relationship, it seems likely the total plasma sheet particle energy density also has a preferred value. This idea is more directly supported by the fact that SME also has a linear relationship with ion precipitation as inferred from global precipitation maps using DMSP data [Newell and Gjerloev, 2011], a relationship which continues smoothly to a near-zero SME values.

A previous work on the distribution of AE values by Vassiliadis et al. [1996] and Consolini and De Michelis [1998] provides a key insight. These authors found that AE is in fact best considered as the sum of two lognormal distributions, one representing quiet times and one active times. In Figure 8, we apply this insight to SME. Indeed, the SME distribution can be
very well approximated as the sum of two lognormal distributions. The “quiet” distribution has a mean of 4.0 \((e^{4.0} = 55 \, \text{nT})\) with \(\sigma = 0.29\). The “active” distribution has a mean of 4.9 \((e^{4.9} = 134 \, \text{nT})\) and \(\sigma = 0.65\). The fitted relative weight of these two distributions is 0.40 for quiet and 0.60 for active.

That relative weighting makes intuitive sense: substorms have a mean recurrence time of about 5 hours, with about 3 hours for the full cycle. Notice that while the peak of the quiet time distribution can be taken to represent something of a ground state for the magnetotail electron population, the active value...
of 134 nT does not represent a substorm peak (which is typically 400–600 nT). Instead, it is the ensemble average over growth, expansion, and especially the prolonged recovery (most of the substorm cycle time is spent in the recovery). [27] Vassiliadis et al. [1996] showed that $AL$ is also the sum of two distributions, each normal in log($AL$). While not finding the same pronounced separation of the quiet and active distribution as those authors did, we essentially confirmed that earlier work (figure not shown). As our results suggest a clearer geophysical meaning for $SME$ than for $SML$, and as our results are substantially similar to that earlier work, we will not further probe the $SML$ distribution here.

[28] There are therefore two well-defined preferred scales for $SME$, one for quiet time and one smeared out value for active time, both of which reflect states of the magnetotail electron population. The characteristic quiet time preferred value of about 5 GW auroral precipitating power can be only maintained if the electron temperature in the magnetotail is maintained well above the few electron volts typical of the solar wind. Indeed, the lower threshold of the DMSP electron detector is 32 eV. Most of the precipitating auroral energy flux comes from keV electrons in the diffuse aurora. This could not continue long without nearly constant driving from the solar wind.

5. Discussion

5.1. Substorm Magnitude, Recurrence, and Solar Wind Driving

[29] We have established that substorms have a characteristic size, typically reaching about 40 GW $AP$ after onset, based on DMSP calibrations (which are a bit lower than the Polar UVI calibrated value). They recur however in a power law distribution, with distinct dynamical regimes for recurrent ($t < 82$ min) and isolated ($t > 3$ hours) substorms. This suggests that the magnetotail can store a finite rather definite amount of magnetic energy, which it releases on whatever timescale is necessary to prevent greatly exceeding that characteristic value for energy storage. That seems to favor certain ideas about substorm instabilities, say those favoring magnetic topology or limits to field line stretching, over others, say those that depend upon particle gradients or particle energy density.

[30] The magnitude of substorms should depend upon both the time since the last substorm (i.e., time over which energy has been stored) and the strength of solar wind driving. Figure 9 places substorms in a grid based on the time since the previous substorm and the intensity of solar wind driving (as based on $d\Phi_{MP}/dt = B_T^{2/3} v^{4/3} \sin^{8/3}(\theta/2)$ [Newell et al., 2007]). The solar wind driving is calculated as a weighted average over the previous 3 hours, based on the results described in the previous work. The color in this plot gives the intensity of the substorm based on $SML$. There is a clear trend for more intense driving to create somewhat more intense substorms. The trend between the time elapsed since the last substorm and the strength of the next substorm is rather weak. However, neither underlying sample correlation is particularly large. The strength of solar wind driving accounts for about 5% ($r = 0.23$) of the strength of a substorm, while the time since the previous substorm is essentially insignificant (well under 1%). The magnitude as well as the precise timing of substorms is thus difficult to predict in any detail.

Figure 9. The intensity of substorms, as measured by SuperMAG $SML$, as a joint function of the time since the previous substorm (y axis) and the intensity of solar wind driving (x axis). The major effect is for stronger substorms to occur under higher driving.
The power output from substorms would be roughly regular if both the magnitude of substorms and their recurrence had preferred values. Yet solar wind driving is well known to be highly irregular. It would technically be possible to have both a preferred magnitude for substorms and a preferred recurrence rate if substorms represented only a minor perturbation to the total energy budget. However, the superposed epoch studies using either imagers or DMSP particles show quite the opposite. Substorms increase global $AP$ by about 70% (over a 2 hour period). As the recovery time is only moderately shorter than the recurrence time, substorms are indeed a major factor in at least particle heating in the magnetotail. Thus, the existence of a preferred substorm size can be taken as an implicit predictor that substorms cannot recur on a preferred timescale.

Finally, we comment briefly on the existence of two distinct regimes for substorm recurrence. For $t < 80–90$ min, $N(\Delta t)$ varies roughly as $1/\Delta t$, while for $t > 3$ hours, $N(\Delta t)$ varies roughly as $1/\Delta t^2$. In the article introducing the SME index [Newell and Gjerloev, 2011], we showed that both isolated substorms and recurrent substorms (the dividing line was put at 2 hours, based on anecdotal reports, as the statistics presented in this article had not been developed) represent the same change in $AP$. Thus, both recurrent and isolated substorms produce increases in diffuse $AP$ of about 8 GW and increases in broadband $AP$ of 1.5 GW, and so on. Yet clearly there is a difference in the dynamical behavior of the magnetosphere, as several figures in this article show, especially Figure 2 (but also Figure 3, and even the power spectra of Figure 6). One explanation is that substorms are a two-step process. Each step may introduce a $1/\Delta t$ factor. For example, the current sheet thins until a near-Earth neutral line forms, which then leads to the explosive release of energy. After the first substorm, the current sheet is still thin, and subsequent substorms may recur without requiring the same step, yet release the same amount of energy. There are other substorm models involving a two-step process, so we are not arguing for one specific model. Rather the present results make it clear that the existence of these two distinct dynamical regimes for the magnetospheric system needs to be accommodated in any physical theory. The magnetosphere “knows” how long it has been since the last substorm, and the dynamics vary accordingly.

### 5.2. Why Does the Magnetotail Never Cool?

The distribution of SME can be resolved as the sum of two distributions each one normal in log(SME), as shown in Figure 8 (and as was previously shown for $AE$ [Vassiliadis et al., 1996; Consolini and De Michelis, 1998]). The peak in the SME distribution occurs at 61 nT, implying about 5 GW $AP$ (the resolved “quiet” peak is slightly smaller). Although the higher activity peak can reasonably be ascribed to a preferred size for substorms, which is comprehensible in the finite ability of the magnetotail to store energy, the existence of the quiet peak shifted away from zero also needs explanation. Solar wind driving as represented by most common estimators does peak at zero, indeed quite strongly so. For example, in the case of interplanetary magnetic field (IMF) $B_z$, fully half the time the driving is zero, and this can be prolonged for a great many hours.

We are aware from examining many years of spectrograms that one rarely, or more likely never, encounters a situation in which the strongest electron precipitation is remotely at solar wind electron values (a few eV). Plasma sheet precipitation at energies of hundreds of electron volts, if not kiloelectron volts, is almost always present. The implication of the distribution in Figures 7 and 8, particularly the absence of small values of SME, is that magnetotail heating appears to be continual, even when there have been many hours without a substorm. The typical plasma sheet electron temperature may dip well below the canonical 1 keV average value, but, at least on a global scale, does not seem to ever drop toward solar wind values (although cold dense patches are certainly encountered within the magnetotail for northward IMF).

Simple calculations and observations demonstrate that hysteresis cannot be the sole explanation, although it can contribute. The loss cone at the equatorial region can be estimated from the ratio

$$\sin^2(\theta_m) = \frac{B_m}{B_{ion}}.$$  \hspace{1cm} (3)

The ratio of magnetotail (“$m$“) to ionospheric (“$ion$“) magnetic fields is roughly 60 nT/6 x 10^4 nT, implying $\theta_m = 1.8^\circ$. Under the common assumption of strong pitch angle scattering maintaining isotropy, 2% of the electron population would be lost with each bounce. As a 1 keV electron travels about 20,000 km/s, this implies about a 2% loss rate every 10 s (a rough bounce time estimate). The actual depletion rate is clearly slower, so electron pitch angle scattering must be less than strong. Still, the recovery from the elevation seen after onset (e.g., as shown by Newell and Gjerloev [2011] or Newell et al. [2010]) implies a cooling time on the order of 2 hours or so. Thus, both calculations and the observation of actual cooling after substorm elevation indicate that hysteresis alone cannot explain why the magnetotail so rarely (if ever) cools.

There are quite relevant similar previous observations, albeit without an explicit discussion of a magnetotail minimum state. Wing et al. [2005, 2006] examined intervals of 10 hours or more northward IMF. That analysis was based on fitting precipitating ion distributions (as well as some in situ Geotail spacecraft data), but the results are certainly reminiscent. Although a cold dense ion population, presumably of magnetosheath origin, increasingly entered the tail for increasing northward IMF, there was very little reduction of the hot ion component, even after 10 hours. It is not just that the magnetotail had not reached zero (cold as magnetosheath) but that the hot component was barely attenuated. This certainly is not possible under the simple picture of the IMF $B_z$ being the major driver of energy into the magnetotail.

It has long been appreciated that frontside magnetopause merging occurs even for northward IMF, especially if $B_z > B_{\infty}$. Figure 9 shows the distribution of several possible driving functions (based on 10 min averages), including $B_z$, $E_{KL}$, $d\Phi_{MP}/dt$, and the Borovsky function [Borovsky et al., 2008]. The latter two are most highly correlated with $AE$ and $AP$ and hence presumably with SME. $d\Phi_{MP}/dt$ is the best empirical fit function, while the Borovsky function has theoretical support. The latter applies the theory of Cassak and Shay [2007] specifically to the frontside magnetopause. In the simple half-wave rectifier view of the magnetosphere...
that considers only $B_z$, the entire half of the distribution with $B_z > 0$ would be a very pronounced maxima at zero driving. As a result, a cold magnetotail with SME peaking near zero should occur based on the $B_z$ distribution.

Neither is there a peak at nonzero values of low driving either for the Kan-Lee electric field ($E_{KL}$) or for the best fit empirical coupling function, $d\Phi_{M} / dt$ [Newell et al., 2007]. Only the Borovsky function shows a peak above zero driving. Even for that function, zero merging is still occasionally predicted. Nonetheless, the magnetotail responds to not just 10 min distributions but many hours. The effect of such averaging is to shift the integrated distribution away from zero driving (zero dayside merging). It is possible that a combination of hysteresis and the Borovsky function could explain why the magnetotail so rarely cools (Figure 10).

Of course, the solar wind might drive the magnetosphere in other ways than frontside merging. Viscous driving is possible, especially at the relatively modest level needed. Merging poleward of the cusp for northward IMF might also drive the magnetotail, preventing it from cooling for quiet times. Figure 8 shows that SME extends to high values (1000 nT and above), much higher than released in most substorms. Conversely, even the best fit distribution with two components (dashed line) overestimates the frequency of very low values of SME (below about $e^{3.0} = 20$ nT). Explaining this full distribution and therefore behavior for global $AP$ and magnetotail electron behavior should be a major goal of space physics research.

6. Summary and Conclusions

The discovery that a global generalization of the $AE$ index, SME, correlates with $AP$ surprisingly well ($r = 0.86$ at a 1 min cadence), and the concomitant ability to routinely identify substorms with acceptable accuracy suggested the value in revisiting issues of magnetosphere and substorms scales and characteristic values.
[41] We find that substorms do not, in fact, recur with a 3 hour, or any other, periodicity, but rather the intervals between substorms form two distinct power law distributions. More specifically, the number of substorms, \( N(\Delta t) \) occurring \( \Delta t \) after the previous varies as \( 1/\Delta t^{-1.19} \) for \( \Delta t < 82 \) min, while for \( \Delta t > 3 \) hours, \( N(\Delta t) \) varies as \( 1/\Delta t^{-1.76} \). The energetic consequences of the recurrent or shorter timescale substorms (as judged by \( AP \) rise and subsequent decay) are essentially indistinguishable from the “isolated” substorms, meaning those which have no precursor (within the previous 2 hours). Nonetheless, two distinct dynamical regimes clearly exist, from the \( N(\Delta t) \) distribution and other evidence. It seems likely that substorms are thus a two-step process, such as current sheet thinning followed by near-Earth neutral x-line formation (to cite one popular possibility). The existence of these two dynamical regimes shows up in many additional results, such as a break in \( SME \) spectral power around 3 hours, and the extent to which the interval between two substorms can predict the time until the next.

[42] In contrast to their power law recurrence pattern, substorms do have a well-defined characteristic magnitude. The large majority of substorms reach a peak 15 min average \( SME \) value of 400–700 nT, with a mean of 656 nT. The high end equates to a little less than 40 GW precipitating \( AP \) (although there is an extended tail to much higher values, up to at least 2000 nT). We argue that if substorms did have characteristic magnitudes in both power dissipated and recurrence intervals, a contradiction would exist: a regular output from irregular solar wind driving. A reasonable interpretation of these results are that the magnetotail can hold a definite and limited amount of magnetic energy, which it releases on whatever timescale is required to prevent excess accumulation.

[43] The time between any two substorms is not a significant predictor of the time until the next substorm. That is, if three consecutive substorms occur at \( t_1, t_2, \) and \( t_3, \) the separation \( \Delta t = t_2 - t_1 \) is a poor predictor of \( t_3 - t_2. \) For recurrent substorms (the \( \Delta t < 82 \) min regime), the correlation is almost trivial (\( r = 0.06 \)). For the longer time regime, the correlation rises markedly to \( r = 0.18, \) but this still predicts only 3.2% of the variance. We interpret this to mean that quasiperiodic sequences of substorms are comparatively unusual.

[44] Because of the intimate relationship between \( SME \) and global \( AP, \) most of which represents diffuse auroral precipitation [Newell et al., 2010], the distribution of \( SME \) 1 min values represents a proxy for the total electron energy in the magnetotail. A previous work on \( AE \) [Vassiliadis et al., 1996] has shown that the distribution can be well approximated by the sum of a “quiet” time and “active” distribution, each of which are normal in log(\( AE \)). The same proves to be true for \( SME \). This implies two distinct characteristic scales for \( AP. \) The “active” component of the \( SME \) distribution can logically be ascribed to the existence of a well-defined typical substorm size and the subsequent prolonged recovery. This component has a relative weighting of 0.60 (60% of the total). The “quiet” component (40% weighting) implies a base \( AP \) of about 4 GW. We argue that nearly all commonly used estimators of solar wind driving of the magnetotail would not produce a finite characteristic quiet time peak. For example, the IMF \( B_z \) is northward half the time, often for many hours (sometimes days), and therefore, a strong peak at zero would be expected from that driving function.

[45] The \( SME \) distribution certainly cannot be reproduced simply with a low-pass filter of any known coupling function. In fact, previous observations and simple calculations suggest that pitch angle scattering and precipitation should be able to cool the magnetotail electrons within hours. Thus, continual driving, not predicted by most estimators of dayside merging, is implied. The frontside merging estimator introduced by Borovsky et al. [2008], based on theoretical work by Cassak and Shay [2007], does have an advantage in seeming to predict that very low frontside merging rates are rare. It is possible that the Borovsky function, which does peak away from zero, along with moderate hysteresis, could account for the finite ground state of 4 GW \( AP. \) The quiet minimum may alternately arise from viscous driving. Regardless, it is reasonable to infer that some energy input from the solar wind into the magnetosphere is nearly always active.

\[ \text{References} \]
Kamide, Y., and G. Rostoker (2004), What is the physical meaning of the \( AE \) index?, Eos Trans. AGU, 85, 188.

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